$$\beta = -\frac{1}{c}\left[\frac{\partial \lambda}{\partial t} + c\, f(t)\right] + f(t)$$

$$\beta = -\frac{1}{c}\frac{\partial \lambda}{\partial t} - \cancel{f(t)} + \cancel{f(t)}$$

$$\boxed{\beta = -\frac{1}{c}\frac{\partial \lambda}{\partial t}.}$$

$$\alpha = \vec{\nabla}\lambda' = \vec{\nabla}\lambda \qquad \left(\because \lambda = \lambda' - c\int_0^t f(t)\,dt \right)$$

time not positions

$\nabla\left(c\int_0^t f(t)\,dt\right) = 0$

Therefore

$$\vec{A}' = A + \alpha.$$

$$\boxed{\vec{A}' = A + \vec{\nabla}\lambda}$$

$$\boxed{V' = V - \frac{1}{c}\frac{\partial \lambda}{\partial t}.}$$

Gauge transformation.

There are the new potential but leaves the fields $\vec{E}$ and $\vec{B}$ unchanged.

Lorentz Gauge :->

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}.$$

but $\vec{B} = \vec{\nabla} \times \vec{A}$, and $\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$

$$\nabla \times (\vec{\nabla} \times \vec{A}) = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial}{\partial t}\left(-\nabla V - \frac{\partial \vec{A}}{\partial t}\right)$$

$$\because \nabla \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - (\vec{\nabla} \cdot \vec{\nabla})\vec{A}.$$

$$= \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$$

$$\vec{\nabla}\cdot(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A} = \mu_0\vec{J} + \mu_0\epsilon_0\left[-\nabla\frac{\partial V}{\partial t} - \frac{\partial^2\vec{A}}{\partial t^2}\right]$$

$$-\nabla^2\vec{A} + \mu_0\epsilon_0\frac{\partial^2\vec{A}}{\partial t^2} = \mu_0\vec{J} - \vec{\nabla}\cdot(\vec{\nabla}\cdot\vec{A}) - \mu_0\epsilon_0\vec{\nabla}\frac{\partial V}{\partial t}$$

$$-\nabla^2\vec{A} + \mu_0\epsilon_0\frac{\partial^2\vec{A}}{\partial t^2} = \mu_0\vec{J} - \vec{\nabla}\left[\vec{\nabla}\cdot\vec{A} + \mu_0\epsilon_0\frac{\partial V}{\partial t}\right]$$

$$\nabla^2\vec{A} - \mu_0\epsilon_0\frac{\partial^2\vec{A}}{\partial t^2} = -\mu_0\vec{J} + \vec{\nabla}\left[\vec{\nabla}\cdot\vec{A} + \mu_0\epsilon_0\frac{\partial V}{\partial t}\right]$$

if $\boxed{\vec{\nabla}\cdot\vec{A} + \mu_0\epsilon_0\frac{\partial V}{\partial t} = 0}$ Lorentz gauge.

Then $$\nabla^2\vec{A} - \mu_0\epsilon_0\frac{\partial^2\vec{A}}{\partial t^2} = -\mu_0\vec{J}$$

Coulomb gauge :-

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{E} = -\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t}$$

$$\vec{\nabla}\cdot\left(-\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t}\right) = +\frac{\rho}{\epsilon_0}$$

$$\nabla^2 V + \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{A}) = -\frac{\rho}{\epsilon_0}$$

$\boxed{\vec{\nabla}\cdot\vec{A} = 0}$ Coulomb gauge.

$$\nabla^2 V = -\frac{\rho}{\epsilon_0}$$ (~~Laplace~~ Poisson equation)

Teacher's Signature

if $\rho = 0$, then $\nabla^2 V = 0$ Laplace equation.

or

$$\vec{\nabla}\cdot\vec{A} = -\mu_0 \epsilon_0 \frac{\partial V}{\partial t} \quad \text{(Lorentz gauge)}$$

$$\nabla^2 V + \frac{\partial}{\partial t}\left(-\mu_0 \epsilon_0 \frac{\partial V}{\partial t}\right) = -\frac{\rho}{\epsilon_0}$$

$$\boxed{\nabla^2 V - \mu_0 \epsilon_0 \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0}}$$

Boundary conditions :->

Those conditions by which fields must be satisfied at the interface of the media are called boundary conditions.

To find electrostatic boundary condition we use two Maxwell equation.

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} \Rightarrow \vec{\nabla}\cdot\vec{D} = \rho \quad (\because \vec{D} = \epsilon_0\vec{E})$$

$$\int \vec{D}\cdot ds = Q$$

$$\vec{\nabla}\times\vec{E} = 0 \qquad \oint \vec{E}\cdot\vec{dl} = 0$$

Dielectric-dielectric boundary condition :->

$$E_1 = E_{1t} + E_{1n}$$

$$E_2 = E_{2t} + E_{2n}$$

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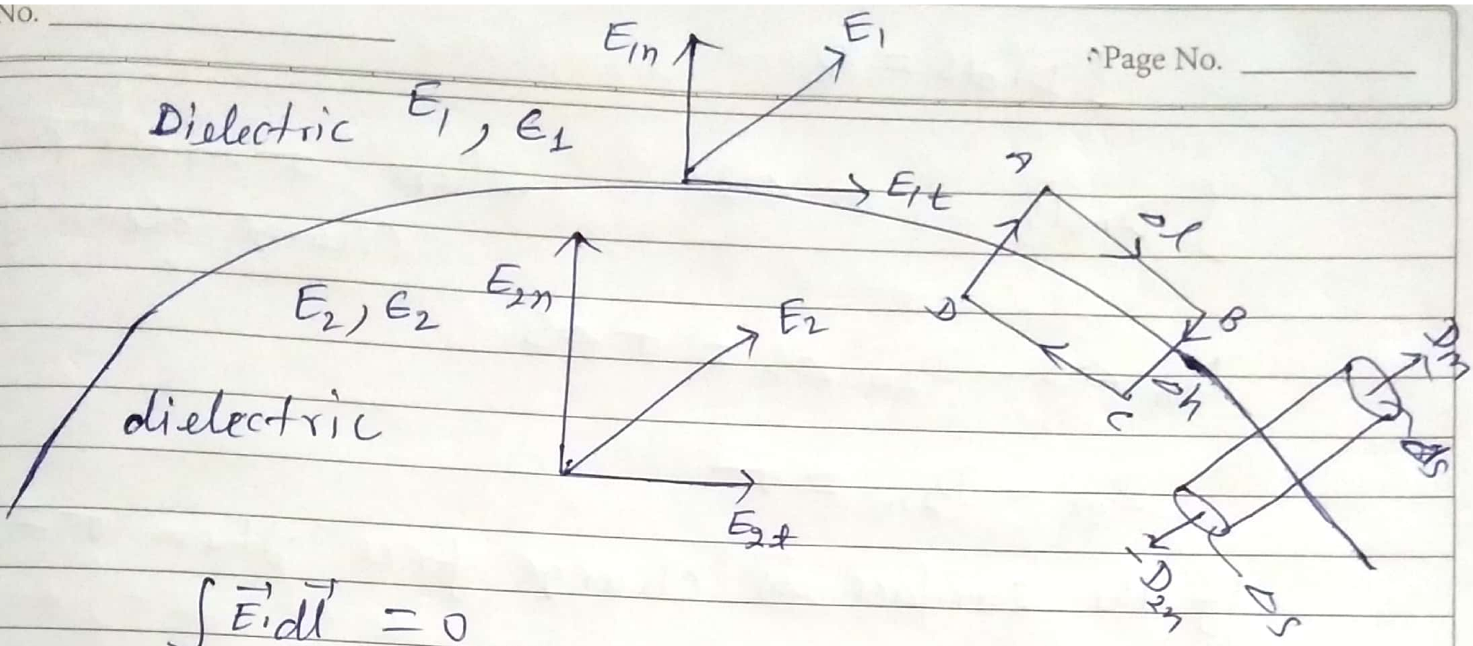

$$\int_l \vec{E}\cdot\vec{dl} = 0$$

$$\int_A^B \vec{E}\cdot\vec{dl} + \int_B^C \vec{E}\cdot\vec{dl} + \int_C^D \vec{E}\cdot\vec{dl} + \int_D^A \vec{E}\cdot\vec{dl} = 0$$

$$E_{1t}\,\Delta l + \cancel{E_{1n}\frac{\Delta h}{2}} - \cancel{E_{2n}\frac{\Delta h}{2}} - E_{2t}\,\Delta l + \cancel{E_{2n}\frac{\Delta h}{2}} + \cancel{E_{1n}\frac{\Delta h}{2}} = 0$$

$$(E_{1t} - E_{2t})\,\Delta l = 0$$

$$\Delta l \neq 0$$

$$E_{1t} - E_{2t} = 0$$

$$\boxed{E_{1t} = E_{2t}}$$

Tangential component of $\vec{E}$ is continuous.

$E_{1t} = E_{2t}$.

$$\boxed{\frac{D_{1t}}{\epsilon_1} = \frac{D_{2t}}{\epsilon_2}}$$

Tangential component of $\vec{D}$ is discontinuous across the boundary.

$$\int \vec{D} \cdot d\vec{s} = Q$$

$$\int \vec{D} \cdot d\vec{s} = \sigma \Delta S$$ where σ is the surface Charge density.

$$D_{1n} \Delta S - D_{2n} \Delta S = \sigma \Delta S$$

$$D_{1n} - D_{2n} = \sigma$$

if the surface is charge free then $\sigma = 0$

$$D_{1n} - D_{2n} = 0$$

$$\boxed{D_{1n} = D_{2n}}$$ Normal component of D is continuous if the surface is Charge free.

$$D_{1n} = D_{2n}$$

$$\epsilon_1 E_{1n} = \epsilon_2 E_{2n}$$

$$\boxed{\frac{E_{1n}}{\epsilon_2} = \frac{E_{2n}}{\epsilon_1}}$$ Normal component of $\vec{E}$ is discontinuous across the boundary.

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